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# The viscosity cross-section for elastic electron-xenon collisions including electron spin polarization

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**Abstract.** The results of the evaluation of the viscosity cross-section for elastic electron-xenon collisions, taking into account the spin-orbit interaction of the continuum electron, in the energy interval from 0.1 eV to 50 eV are presented and discussed. The calculations are performed on the basis of three theoretically derived sets of phase shift data obtained by different authors and on the deduced relativistic expression for the viscosity cross-section in terms of phase shifts discerning the spin-up and spin-down states of the scattered electrons. Comparison with viscosity cross-sections, as evaluated from non-relativistic phase shifts extracted from experiments, strongly favours the relativistic results. The assumption of isotropic scattering is critically examined and the error induced by its use is shown to persist to the same extent as in non-relativistic calculations, at least in the energy region considered.

**PACS.** 34.80.Bm Elastic scattering of electrons by atoms and molecules

## 1 Introduction

In the experiment and theory of electron-atom collisions the observed spin polarization (or depolarization, *i.e.* change in polarization) of initially unpolarized (polarized) electrons after being scattered by the target atom, has gained increasing attention in the study of spin-dependent interactions [1–6].

In the case of elastic electron scattering from heavy targets in the ground state and without spin, such as the heavier rare-gas atoms and xenon ($5p^6\ {}^1\mathrm{S}_0$) in particular, the spin-polarization effect is caused by the spin-orbit interaction of the continuum electron in the field of the target atom with zero angular momentum. For this type of collision process experiments have been achieved [1,6] yielding a complete set of measured observables (namely, the three polarization parameters and the absolute differential cross-section), that contain the maximum possible information about the scattering as described by the quantum-mechanical complex scattering amplitudes [6,7]. These can be adequately evaluated *ab initio* within the relativistic formalism and, in particular, from the Dirac equations which describe the electron spin and the spin-orbit coupling [8,9]. Thus a valuable link between the experiment and the theory is established, resulting in a new thrust in this area of collision physics. Much of the intensive research under way is devoted to the xenon target, as both its high atomic number $Z$ and closed-shell structure make it suitable for the study of the spin-orbit interaction by way of the spin-polarization effect [3,4,10,11]. In these studies the low-energy region, *i.e.* up to around 50 eV, is of special interest [10–12] since, along with the electron spin-orbit coupling, other interactions (*e.g.*, the polarization of the atomic charge cloud by the electric field of the slowly moving electron projectile) have to be carefully considered there. For elastic low-energy electron-xenon (e-Xe) scattering the spin-orbit interaction is large enough to have its effect not only on the differential cross-section, but also on the cross-sections integrated over the angles, such as the total elastic $Q_\mathrm{t}$ and momentum-transfer $Q_\mathrm{m}$ cross-sections [12]. It is well established that the spin-orbit interaction, as described by the Dirac equations, alters noticeably both $Q_\mathrm{t}$ and $Q_\mathrm{m}$ values, as compared with the corresponding ones deduced on the basis of the spin-independent Schrödinger equation. The common feature of these changes is the displacement of the Ramsauer-Townsend (RT) minimum to somewhat higher energies, and the increase (decrease) of the cross-section values for energies below (above) the RT minimum [9,12–14], resulting in an improved agreement between theory and measurements.

As far as the elastic electron-atom/molecule scattering is concerned, reliable data for integral cross-sections, other than $Q_\mathrm{t}$ and $Q_\mathrm{m}$, are of significant interest as well, as these also determine the electron transport properties in a variety of weakly ionized plasmas, gas discharges and electron swarms. More precisely, the higher-order cross-

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sections, generally defined [15] as

$$Q_{(r)} = 2\pi \int_0^\pi \sigma_{\mathrm{ea}}^{\mathrm{el}}(\varepsilon, \theta)[1 - P_r(\cos\theta)] \sin\theta \mathrm{d}\theta \quad (r = 1, 2, \dots), \tag{1a}$$

where $\sigma_{\mathrm{ea}}^{\mathrm{el}}(\varepsilon, \theta)$ is the differential cross-section for elastic electron-atom collisions and $P_r(\cos\theta)$ are the Legendre polynomials, or alternatively [16] as

$$Q^{(r)} = 2\pi \int_0^\pi \sigma_{\mathrm{ea}}^{\mathrm{el}}(\varepsilon, \theta)(1 - \cos^r\theta) \sin\theta \mathrm{d}\theta \quad (r = 1, 2, \dots) \tag{1b}$$

appear in any kinetic modelling based on the multiterm expansion of the electron energy distribution function (EEDF) with respect to Legendre polynomials. It is easily seen that for anisotropic scattering (*i.e.* for $\sigma_{\mathrm{ea}}^{\mathrm{el}}$ *dependent* on $\theta$), all $Q_{(r)}$ (or $Q^{(r)}$) with $r \neq 1$ differ from $Q_{(1)} (\equiv Q^{(1)} \equiv Q_{\mathrm{m}})$, and that they all converge to the total cross-section $Q_{\mathrm{t}}$ for large values of $r$.

In this paper the attention is focused on the viscosity cross-section $Q_{\mathrm{v}} \equiv Q^{(2)}$ $(r = 2)$ taken, according to (1b), as

$$Q_{\mathrm{v}} = 2\pi \int_0^\pi \sigma_{\mathrm{ea}}^{\mathrm{el}}(\varepsilon, \theta)(1 - \cos^2\theta) \sin\theta \mathrm{d}\theta\,, \tag{2}$$

and arising wherever the standard two-term (Lorentz) approximation is insufficient and, therefore, replaced by the EEDF expansions with *three* or more terms retained. The multiterm approximation (with $r$ ranging from 3 up to 6 or 8) has been extensively used for almost two decades [17, 18] in accurate evaluations of the EEDF and of the related electron transport parameters, especially if, in addition to the presence of external electric field and inelastic (along with the elastic) collisions, a spatial or temporal relaxation process takes place [19,20]. However, the anisotropy of the scattering itself, although evident, is often neglected in these evaluations without other justification than the simplification it entails. The deep RT minima apparent in the integral cross-sections for electron scattering in rare gases of larger $Z$, in molecular gases like methane or in mixtures of both, prevent successful randomization of the electron velocities, especially if an inelastic process takes place at energies around these minima which thereby contribute to the anisotropy of the system in general and to that of the EEDF in particular.

The distinction of higher-order cross-sections is also important in deducing the cross-section data from electron transport parameters measured in swarm experiments. The standard modified effective range theory (MERT) [21], widely applied for the deduction of low-energy $Q_{\mathrm{m}}$ cross-sections, is extended [22] to yield sets of higher-order cross-sections (*e.g.*, up to $r = 3$, *i.e.* including $Q_{\mathrm{v}}$, in [22]) which reproduce not only the measured electron drift velocity and transverse diffusion coefficient, but the measured *longitudinal* diffusion coefficient as well. (In fact, measurements of the longitudinal diffusion coefficient have served as the basis for experimental determination of the viscosity cross-section.)

Low-energy electron scattering from xenon deserves attention from both the above-mentioned aspects. Xenon as a high-$Z$ target induces electron spin-orbit coupling large enough to manifest in integral cross-sections and as a rare gas is much exploited in different types of low-temperature discharges important in technological applications (*e.g.*, radiation detectors, efficient Xe-halide light sources). A comprehensive study of underlying kinetics of the basically non-equilibrium, collision-dominated plasmas placed in external electric fields requires, *inter alia*, a detailed knowledge of the e-Xe cross-sections, including those of higher order.

The viscosity cross-section for elastic e-Xe collisions was evaluated previously [23] within the electron energy interval 0.1 eV–54.4 eV. The evaluations were based on different sets of phase shift data obtained from both the non-relativistic *ab initio* calculations and the procedures fitting to differential cross-sections measured in beam experiments. It was found that the assumption of isotropic scattering (which assumes that $\sigma_{\mathrm{ea}}^{\mathrm{el}}$ is *independent* of $\theta$ and, therefore, equates $Q_{\mathrm{t}}$ and $Q_{\mathrm{m}}$ and sets $Q_{\mathrm{v}}/Q_{\mathrm{m}} = 2/3$) can induce errors in the actual $Q_{\mathrm{v}}$ cross-section values as high as 40% to 60%.

The aim of the present paper is to examine whether and to what extent the spin-orbit interaction of the electrons elastically scattered from xenon influences the viscosity cross-section $Q_{\mathrm{v}}$ as defined by (2), and to provide low-energy (0.1 eV–50 eV) viscosity cross-section data as a complement to abundant evaluations of the relativistic total and momentum-transfer cross-sections for elastic e-Xe collisions in this energy region. The present evaluations of $Q_{\mathrm{v}}$ are based on three sets of the phase shift data for e-Xe elastic scattering previously determined from the solution of the relativistic scattering equation by different authors [24–26], and on the relativistic expression for $Q_{\mathrm{v}}$ which accounts for different electron spin directions after scattering [27].

The results arrived at, as compared with $Q_{\mathrm{v}}$-values deduced from experiments [28,29], indicate distinct improvement of the relativistic over non-relativistic calculations for the viscosity cross-section.

## 2 Basic data and evaluation procedure

The relativistic solution of the scattering problem yields two different phase shifts $\delta_l^+(k)$ and $\delta_l^-(k)$, for the spin-up $(j = l + 1/2)$ and spin-down $(j = l - 1/2)$ states, respectively, which correspond to each orbital angular momentum $l$, $l \neq 0$ and are functions of the electron linear momentum $k$. (In what follows atomic units are used, except for the final cross-section results which are reported in units $10^{-20}$ m$^2$ as functions of energy in eV, $\varepsilon(\mathrm{eV}) = 13.6058\, k^2$.) In terms of these phase shifts the two complex scattering amplitudes, *i.e.* the direct $f(\theta, \delta_l^+, \delta_l^-)$ and the spin-change $g(\theta, \delta_l^+, \delta_l^-)$ ones (*e.g.*, [24]), are given by

$$f(\theta,\delta_l^+,\delta_l^-)=\frac{1}{2ik}\sum_{l=0}^{\infty}\{(l+1)[\exp(2i\delta_l^+)-1] \\ +l[\exp(2i\delta_l^-)-1]\}P_l(\cos\theta)\,, \quad (3)$$

and

$$g(\theta,\delta_l^+,\delta_l^-)=\frac{1}{2ik}\sum_{l=0}^{\infty}[\exp(2i\delta_l^-)-\exp(2i\delta_l^+)]P_l^1(\cos\theta)\,, \quad (4)$$

where $P_l(\cos\theta)$ and $P_l^1(\cos\theta)$ denote the Legendre and the associated Legendre polynomials, respectively. The elastic differential scattering cross-section for the unpolarized incident electron beam is then

$$\sigma_{\rm ea}^{\rm el}(k,\theta)=\left|f(\theta,\delta_l^+,\delta_l^-)|^2+|g(\theta,\delta_l^+,\delta_l^-)\right|^2\,. \quad (5)$$

With the aid of equations (3-5) one can express the integral cross-sections (namely, the total elastic $Q_{\rm t}$, the momentum-transfer $Q_{\rm m}$ and all the higher-order cross-sections) in terms of the phase shifts $\delta_l^+$ and $\delta_l^-$, and the electron linear momentum $k$ (*i.e.* electron energy $\varepsilon$). By integrating over the scattering angle $\theta$, in accordance with (2) for the viscosity cross-section $Q_{\rm v}$ in particular, one obtains [27] the following sum over $l$:

$$Q_{\rm v}=\frac{4\pi}{k^2}\sum_{l=0}^{\infty}\left[\frac{(l+1)(l+2)(l+3)}{(2l+3)(2l+5)}\sin^2(\delta_{l+2}^+-\delta_l^+)\right. \\ +\frac{l(l+1)(l+2)}{(2l+1)(2l+3)}\sin^2(\delta_{l+2}^--\delta_l^-) \\ +\frac{2(l+1)(l+2)}{(2l+1)(2l+3)(2l+5)}\sin^2(\delta_{l+2}^--\delta_l^+) \\ \left.+\frac{2l(l+1)}{(2l-1)(2l+1)(2l+3)}\sin^2(\delta_l^+-\delta_l^-)\right]\,. \quad (6)$$

The spin-orbit interaction is the most remarkable with the lower partial waves (the *p*- and *d-wave* in particular) and its significance tends to diminish with increasing $l$, the differences between $\delta_l^+$ and $\delta_l^-$ becoming practically negligible ($\delta_l^+\approx\delta_l^-=\delta_l$) for comparatively low values of $l$, *i.e.* $l\geq 6$. However, if the spin-orbit interaction is neglected altogether (non-relativistic approach), the spin-up and spin-down phase shifts merge (for all values of $l$) into one, $\delta_l^+=\delta_l^-\equiv\delta_l$, the spin change amplitude $g$ vanishes and the expression for $Q_{\rm v}$ assumes the well-known non-relativistic form [30]

$$Q_{\rm v}^{\rm (nr)}=\frac{4\pi}{k^2}\sum_{l=0}^{\infty}\frac{(l+1)(l+2)}{(2l+3)}\sin^2(\delta_{l+2}-\delta_l)\,. \quad (7)$$

The present evaluations are based on three independent, theoretically predicted complete sets of data on relativistic phase shifts for the elastic scattering of electrons from xenon atoms. These sets of data are: (*a*) reported by Sin Fai Lam [24], (*b*) derived by Sienkiewicz and Baylis [25], and (*c*) obtained by McEachran and Stauffer [26]. The choice of these particular phase shift data sets was based on the reliability with which they appear to reproduce the results of some measurements of e-Xe $Q_{\rm t}$ and $Q_{\rm m}$ cross-sections [31–33], including some recent ones [2, 34]. All the three selected sets $\{\delta_l^{\pm}(\varepsilon)\}$ were derived within the frame of the relativistic Hartree-Fock (Dirac-Fock) theory. Each pertains to a somewhat different interval of incident electron energies $\varepsilon$ (linear momentum $k$) and extends over a different range of angular momenta $l$, from $l=0$ up to $l=l^*$ ($l^*$ denotes the highest angular momentum number for which the phase shifts $\delta_{l^*}^{\pm}$ or $\delta_{l^*}$ were reported).

The evaluation of the set of phase shift data (*a*) ($\varepsilon=$ 0.01–30 eV, $\delta_l^{\pm}$ for $l=0,1,2$ and $\delta_l$ for $l=3,\dots,8$) was performed in the static exchange approximation, the static and relativistic potentials included. The exchange interaction was taken into account only as described by the large components of the bound-state orbitals and, consequently, the Dirac equations were reduced to one Schrödinger-type equation for the continuum wave function. The effects of target polarization were taken into account by assuming the Pople-Schofield polarization potential. However, the one-electron terms in the exchange kernel have been neglected.

The sets of phase shift data (*b*) and (*c*) were both evaluated by solving the Dirac equations, with relativistic and polarization potentials, and exchange included for both the large and small components of the continuum wave function. (The method applied in obtaining data (*c*) is analogous to the one described in [35] with the difference that the phase shifts were obtained by solving the Dirac equations directly [13] rather than by solving the corresponding approximate relativistic Schrödinger equation, as was done in [35].) Therefore, the main differences between the two approaches that led to phase shift data sets (*b*) and (*c*) apparently arise from the different treatment of the polarization interaction between the target and the incident electron. In obtaining the set of phase shifts (*b*) ($\varepsilon=0.4$–30 eV, $\delta_l^{\pm}$ for $l=0,\dots,5$) all electrons were treated as equivalent (*i.e.* indistinguishable), their interaction being described by a single model polarization potential with one adjustable parameter. Moreover, additional distortion of the atomic charge cloud due to exchange interaction, *i.e.* the exchange polarization, was also taken into account. In determining the set of phase shifts (*c*) ($\varepsilon=0.1$–50 eV, $\delta_l^{\pm}$ for $l=0,\dots,11$) the non-relativistic *ab initio* polarized-orbital method was adopted, and in the expansion of the polarization potential in multipoles only the dipole part was retained [35].

The authors of (*a*) and (*b*) reported also the $Q_{\rm t}$ and $Q_{\rm m}$ cross-sections that follow from their respective phase shift data. Analogous evaluations of $Q_{\rm t}$ and $Q_{\rm m}$ were performed on the basis of (*c*) (and repeated for sets (*a*) and (*b*)) in the course of the present work; the results pertaining to (*a*) and (*b*) were in remarkable agreement with the cross-section values previously reported by the respective authors.

The viscosity cross-section values $Q_v$ for elastic e-Xe collisions reported in this paper apply to incident electron energies from 0.1 eV to 50 eV. This energy range, common (at least in part) to the selected sets of phase shift input data, embraces both the RT minimum and the subsequent maximum in $Q_v$ (as well as in $Q_m$ and in $Q_t$). It is also the electron energy range within which the peculiarities of elastic e-Xe collisions are known to be dominant for the macroscopic behaviour of different low-temperature, low-pressure discharges both in pure xenon, and in mixtures of xenon with other rare gases [20,36].

The present calculations of $Q_v$ are carried out by applying the relativistic formula (6) to the phase shift sets ($a$), ($b$) and ($c$) up to $l = l^*$. All the higher phase shifts, *i.e.* for $l > l^*$, are accounted for by the non-relativistic *two-term* formulae of Ali and Fraser [37] (*i.e.* up to order $k^4$ in the perturbation expansion, with the quadrupole polarizabitily set to zero):

$$\tan\delta_l = k^2\beta a_l + k^4\beta^2 b_l \quad (l > l^*)\,, \tag{8}$$

where $\beta$ is the dipole polarizability of the xenon atom taken to be 27.06, and $a_l$ and $b_l$ are coefficients dependent upon $l$ and given by Ali and Fraser [37]. By the use of (8) the summations in (6) or (7) can be extended up to the required accuracy. Presently, the truncation of the virtually infinite and convergent sum (6) is safely performed at $l = 100$. As was shown on the basis of analytical estimates [27], truncating the sum at $l = 100$ instead of extending it up to infinity induces a relative error in the cut-off rest of the sum of about $10^{-6}$ with $l^* = 11$ (data ($c$)), and even less with $l^* = 8$ and $l^* = 5$ (data ($a$) and ($b$), respectively).

## 3 Results and discussion

The obtained values for the viscosity cross-section for elastic e-Xe collisions are listed in Table 1 and presented in Figure 1. The results arrived at on the basis of the different data, ($a$) (dot-dashed curve), ($b$) (full light curve) and ($c$) (bold curve), are seen to be in reasonable overall agreement. In Figure 1 they are also compared with the three sets of $Q_v$-values reported earlier [23]. The theoretical set of values (represented by the light dotted curve) covering the whole energy interval considered pertains to phase shifts deduced previously by McEachran and Stauffer [38], within the non-relativistic approach that parallels their relativistic one adopted for the extraction of data ($c$). Two discrete sets of points (Fig. 1) correspond to $Q_v$'s that follow from two sets of phase shift data obtained by different *non-relativistic* analysis techniques of mutually independent *measurements* of e-Xe elastic differential cross-sections. The full triangles (at and below 1 eV) belong to the MERT parameters derived by Weyhreter *et al.* [28]. The error bars on the respective $Q_v$'s are estimated presently on the basis of uncertainties in the first two phase shifts $l = 0, 1$ (for which analytical expressions as a function of energy and of fitting parameters are provided by MERT) as induced by the uncertainty

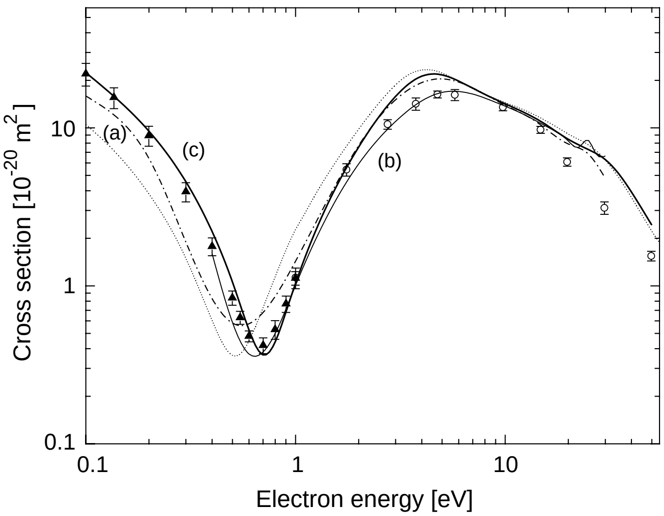


**Fig. 1.** Viscosity cross-section for elastic e-Xe collisions; curves: dot-dashed ($a$) - Sin Fai Lam 1982, full ($b$) - Sienkiewicz and Baylis 1989, bold ($c$) - McEachran and Stauffer 1998; *light dotted* - McEachran and Stauffer 1984; points: full triangles (▲) - Weyhreter *et al.* 1988, hollow circles (○) - Register *et al.* 1986.

in the energy scale reported by Weyhreter *et al.* to be ±15 meV. In the energy region from 1 eV and above, the point-plotted $Q_v$-values (hollow circles) follow from phase shifts extracted by Register *et al.* [29]. Their estimate of the overall relative error in normalized cross-sections (including error in normalization, error in phase shifts and systematic error) at each energy is taken to apply to the $Q_v$-values, too.

Figure 1 reflects the changes in the theoretically deduced $Q_v$-values when according to (6) the spin polarization of scattered electrons is taken into account. Main features of the present $Q_v$'s, as compared with the non-relativistic theoretical results (much in analogy to $Q_t$ and $Q_m$ [9,13]), are the displacement of the RT minimum (located according to non-relativistic predictions at around 0.5 eV) to somewhat higher energies (most markedly, for around 0.2 eV, with data ($c$)), and an increased minimum cross-section value (a noticeable increase of about 60% is seen with data ($a$)). According to all the three sets of data ($a$), ($b$) and ($c$), the relativistic $Q_v$'s are considerably above the non-relativistic ones [23] for energies below the RT minimum; not less than 20% with data ($a$), and even by a factor of above 2 to around 3.6 with data ($c$). Quite to the contrary, they are appreciably lower (as much as 55% with data ($b$) and ($c$)) than the non-relativistic values in the energy interval above the RT minimum and including the maximum. According to data ($a$), ($b$) and ($c$) the maximum in $Q_v$, as compared to the non-relativistic result, is displaced to somewhat higher energies with a decreased maximum cross-section value; most noticeably, around 25%, with data ($b$).

The two sets of phase shift data deduced by McEachran and Stauffer, *i.e.* the one used presently ($c$) and the non-relativistic one [38], are used as the most

**Table 1.** Viscosity cross-section (in $10^{-20}$m$^2$) for elastic e-Xe collisions from phase shift data by ($a$) Sin Fai Lam 1982, ($b$) Sienkiewicz and Baylis 1989, ($c$) McEachran and Stauffer 1998.

| $\varepsilon$(eV) | $Q_v$ ($10^{-20}$ m$^2$) | | |
|---|---|---|---|
| | ($a$) | ($b$) | ($c$) |
| 0.1 | 15.92 | | 22.36 |
| 0.2 | 6.41 | | 9.52 |
| 0.3 | | | 4.56 |
| 0.4 | | 1.60 | 2.21 |
| 0.5 | 0.581 | | 1.06 |
| 0.6 | | 0.370 | 0.526 |
| 0.7 | | | 0.365 |
| 0.8 | | 0.430 | 0.442 |
| 0.9 | | | 0.681 |
| 1 | 1.43 | 1.01 | 1.04 |
| 2 | 7.74 | 5.92 | 7.56 |
| 3 | | 10.99 | 15.83 |
| 4 | | 14.84 | 21.25 |
| 5 | 20.43 | 16.77 | 21.65 |
| 5.5 | | 17.04 | |
| 6 | | | 19.78 |
| 6.5 | | 16.74 | |
| 7 | | | 17.83 |
| 7.5 | | 15.91 | |
| 8 | | | 16.30 |
| 9 | | 14.52 | 15.13 |
| 10 | 13.89 | 13.72 | 14.20 |
| 12.5 | | 12.00 | |
| 14 | 10.97 | | |
| 15 | | 10.55 | 10.85 |
| 17.5 | | 9.45 | |
| 20 | 7.90 | 8.26 | 8.47 |
| 22.5 | | 7.56 | |
| 25 | 6.80 | 8.32 | 7.27 |
| 27 | | 6.91 | |
| 30 | 4.79 | 6.64 | 6.29 |
| 35 | | | 5.06 |
| 40 | | | 3.93 |
| 45 | | | 3.05 |
| 50 | | | 2.42 |

suitable to compare the deviations of the relativistic with respect to the non-relativistic results pertaining to the first three ($i.e.$ $Q_t, Q_m$ and $Q_v$) integral cross-sections. For all three cross-sections the largest deviations on the energy interval examined occur around the RT minimum. It is interesting to note that within this energy range the deviations are the smallest in $Q_t$, and much more significant in $Q_m$ and $Q_v$: being larger in $Q_m$ than in $Q_v$ for energies below and smaller in $Q_m$ than in $Q_v$ for energies above the RT minimum.

As compared with the cross-section values pertaining to experiments, phase shift data ($c$) seem to reproduce the RT minimum most satisfactorily. In the low-energy region up to 2 eV, the $Q_v$-values from data ($c$) fall into the error bars, or are slightly above the upper limit of the error bars, on the $Q_v$-values pertaining to the experimental data of Weyhreter *et al.* [28] and Register *et al.* [29]. With respect to these experimental values, the $Q_v$-values deduced from data ($b$) show similar behaviour, though in the energy range immediately following the RT minimum and including the maximum, up to around 10 eV. Excellent agreement between the four $Q_v$-values, two theoretical (pertaining to data ($b$) and ($c$)) and two experimental [23,28,29], is achieved at 1 eV. Thereafter up to some 15 eV good agreement between the $Q_v$'s that follow from data ($a$) and ($c$) is also seen.

In the intermediate energy region, between the RT minimum and the maximum, the dominant contribution to the integral cross-sections comes from the *d-wave* phase shift, the $Q_v$ cross-section (6) being particularly sensitive to the changes in the *d-wave* phase shift [23]. The relativistic decrease of the *d-wave* phase shift leads to lower values of $Q_v$, and this is still more noticeable if the exchange *due* to the *polarization* interaction is included, as shown by data ($b$). Thus the $Q_v$'s that follow from data ($b$) are the lowest in the energy range considered while the ones pertaining to data ($a$) and data ($c$), obtained within the *static* and *adiabatic* exchange approximations, respectively, fall approximately midway between the former and the non-relativistic values. Good agreement between the results that follow from the relativistic ($c$) and the non-relativistic approach of McEachran and Stauffer [38] can be noticed practically over the whole energy interval above the maximum up to 50 eV. This is not surprising since, as pointed out by those authors [35], the higher partial-wave phase shifts, dominant in this energy range, show diminishing splitting when evaluated relativistically and for $l = 6$ are effectively quite close to the non-relativistic ones [26,38]. However, at the high-energy tail substantial discrepancies are noticed with respect to the values predicted from the experimental data of Register *et al.* [29], clearly indicating the energy range (20–50 eV) in which the general overestimate [14,24] of theoretically evaluated integral cross-sections (including $Q_t$ and $Q_m$) is yet to be accounted for.

The importance of *strict* relativistic calculations for e-Xe integral cross-sections is well illustrated by the sets of theoretical phase shift data selected presently as they pertain not only to modified versions of the scattering equation, but also to different potentials used to account for the correlation effects between the scattered electron and the target electrons. The $Q_v$-values that follow from phase shifts deduced from the measurements clearly favour the relativistic results. These are in much better agreement with experiment if the phase shifts are determined from the solution of the exact relativistic Dirac scattering equations and with the exchange polarization taken into account, rather than from the corresponding relativistic Schrödinger equation and within the static exchange approximation.

As far as the error introduced by the assumption of isotropic scattering is concerned, it is worthwhile noticing that it persists with relativistic calculations to roughly the same extent as with the non-relativistic ones [23]. The

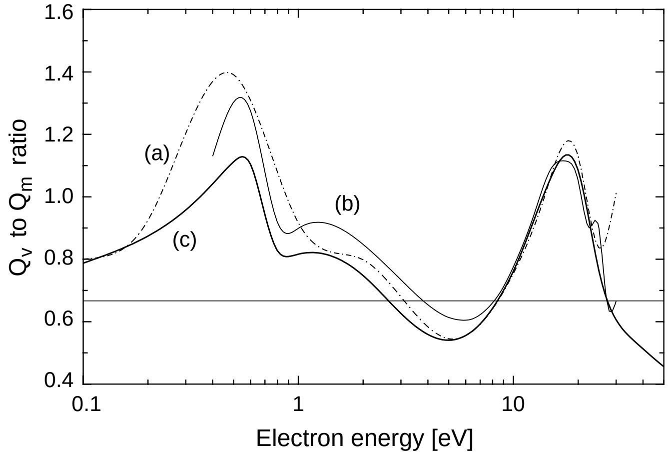


**Fig. 2.** $Q_\mathrm{v}/Q_\mathrm{m}$ ratio for elastic e-Xe collisions; curves: dot-dashed (*a*) - Sin Fai Lam 1982, full (*b*) - Sienkiewicz and Baylis 1989, bold (*c*) - McEachran and Stauffer 1998; *light horizontal line*: isotropic scattering value, $Q_\mathrm{v}/Q_\mathrm{m} = 2/3$.

prominent peaks in the $Q_\mathrm{v}/Q_\mathrm{m}$ ratio in Figure 2 agree fairly well in that they indicate the energy regions where the assumption of isotropic scattering underestimates the actual $Q_\mathrm{v}$-values by as much as 50% (data (*a*)) in the region of the RT minimum and 40% (data (*a*), (*b*) and (*c*)) in the energy region of about 15 to 25 eV. The same assumption, however, leads to an overestimation of the actual $Q_\mathrm{v}$-values by about 20% (data (*a*) and (*c*)) in the vicinity of the maximum, and by even 45% (data (*c*)) in the high-energy tail.

The distinctly improved agreement between the $Q_\mathrm{v}$-values derived from the phase shift data deduced from measurements and the ones presently evaluated unquestionably supports the relativistic approach in determining the viscosity cross-sections for elastic e-Xe scattering in the energy interval considered. If relativistic phase shifts are used as standard to reproduce accurate $Q_\mathrm{t}$- and $Q_\mathrm{m}$-values, there is no reason not to apply them to the evaluation of $Q_\mathrm{v}$ too. The more so since the deviations of the relativistic with respect to non-relativistic cross-section values in the low-energy region are significantly larger in both $Q_\mathrm{m}$ and $Q_\mathrm{v}$ than in $Q_\mathrm{t}$. Thus evaluated (by avoiding, *inter alia*, the unreliable assumption of isotropic scattering) the $Q_\mathrm{v}$'s are likely to prove useful in the study of e-Xe low-temperature kinetics, given the general consensus that it strongly depends on the accurate data for the collision processes involved.

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